

Two port network

Q. Define Z-parameters (Impedance parameters) (open circuit impedance parameters)

Ans: These are also called impedance parameters. These are obtained by expressing voltages at two ports in terms of currents at two ports. Thus I_1 & I_2 are independent variables, while V_1 & V_2 are dependent variables. Thus we have

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

In matrix form
$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

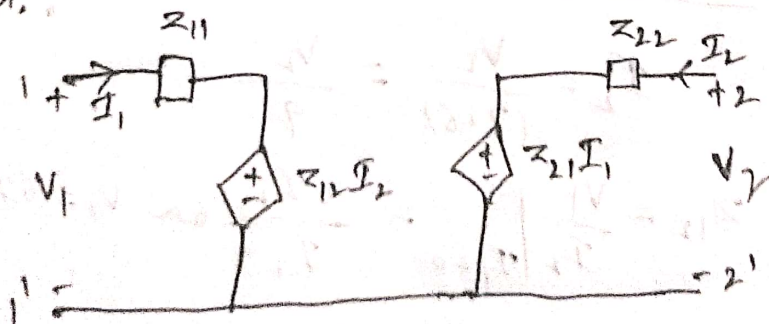
$$Z_{11} = \left. \frac{V_1}{I_1} \right|_{I_2=0} \quad (\Omega)$$

$$Z_{12} = \left. \frac{V_1}{I_2} \right|_{I_1=0} \quad (\Omega)$$

$$Z_{21} = \left. \frac{V_2}{I_1} \right|_{I_2=0} \quad (\Omega)$$

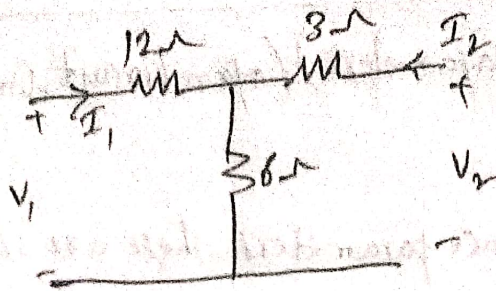
$$Z_{22} = \left. \frac{V_2}{I_2} \right|_{I_1=0} \quad (\Omega)$$

These parameters are defined only when the current in one of the ports is zero. This corresponds to the condition that one of the ports is open circuited. Hence Z-parameters are named as open circuit impedance parameters.

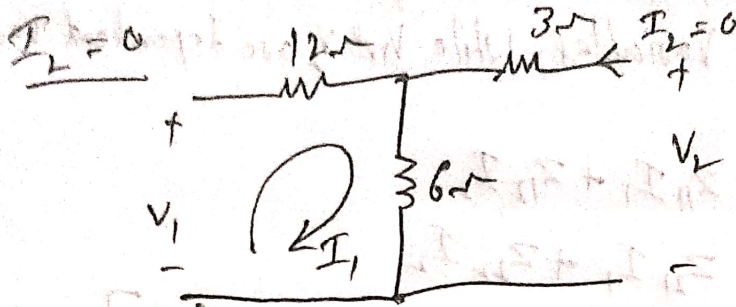


Equivalent network of a two port network in terms of Z-parameters

Q. Find the Z-parameters for the Circuit shown in the figure.



Sol:

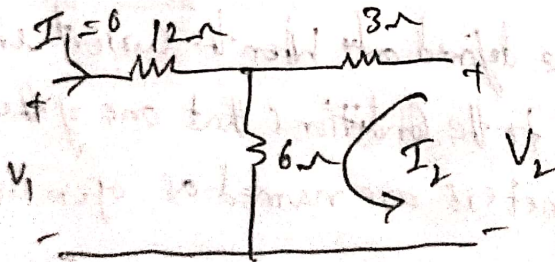


$$I_1 = \frac{V_1}{(12+6)} = \frac{V_1}{18}$$

$$Z_{11} = \left. \frac{V_1}{I_1} \right|_{I_2=0} = \frac{V_1}{\frac{V_1}{18}} = 18 \Omega$$

$$Z_{21} = \left. \frac{V_2}{I_1} \right|_{I_2=0} = \frac{6I_1}{I_1} = 6 \Omega \quad V_2 = 6I_1$$

$I_1 = 0$



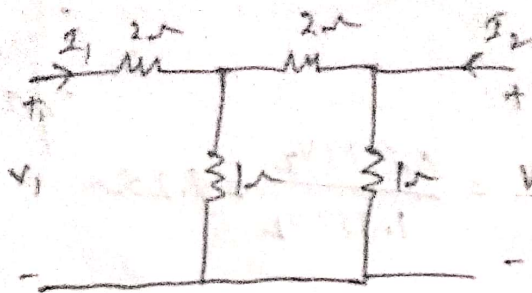
I_2 should always be taken in anticlockwise direction.

$$I_2 = \frac{V_2}{(3+6)} = \frac{V_2}{9}$$

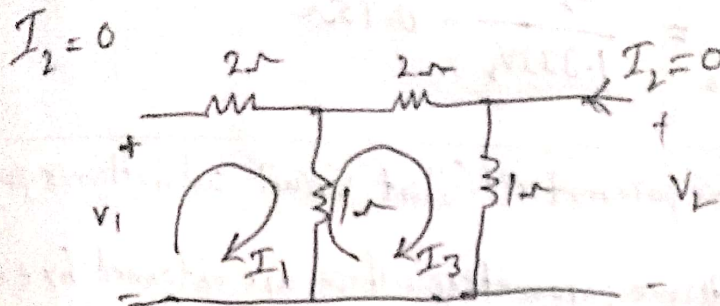
$$Z_{12} = \left. \frac{V_1}{I_2} \right|_{I_1=0} = \frac{6I_2}{I_2} = 6 \Omega \quad V_1 = 6I_2$$

$$Z_{22} = \left. \frac{V_2}{I_2} \right|_{I_1=0} = \frac{V_2}{\frac{V_2}{9}} = 9 \Omega$$

Q. for the network shown in the figure determine the open circuit impedance parameters.



Sol:



$$(3) I_1 + (-1) I_3 = V_1$$

$$(-1) I_1 + (4) I_3 = 0$$

In calculator for
C₁ = 1 (Coefficient of
V₁)

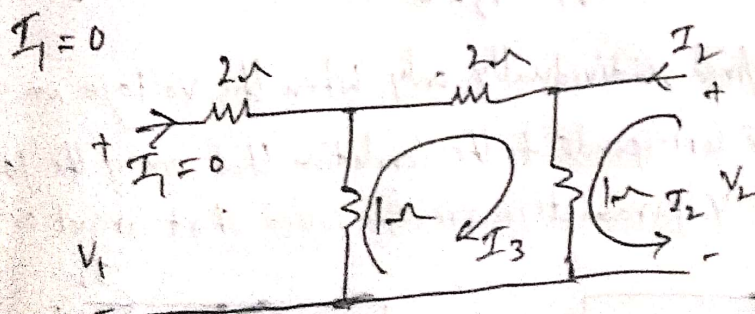
Attach V₁ in answers
for I₁ & I₃

$$I_1 = 0.3636 V_1$$

$$I_3 = 0.090 V_1$$

$$Z_{11} = \frac{V_1}{I_1} \bigg|_{I_2=0} = \frac{V_1}{0.3636 V_1} = 2.75 \Omega$$

$$Z_{21} = \frac{V_2}{I_1} \bigg|_{I_2=0} = \frac{1 I_3}{I_1} = \frac{0.090 V_1}{0.3636 V_1} = 0.247 \Omega \quad V_2 = 1 I_3$$



$$(4) I_3 + (+1) I_2 = 0$$

$$(+1) I_3 + (1) I_2 = V_2$$

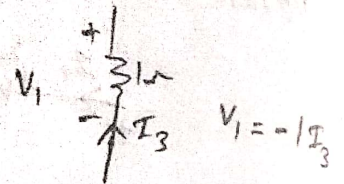
off-diagonal elements
are +ve. \because I₃ & I₂
are in the same direction
in 1Ω

$$I_3 = -0.333 V_2$$

$$I_2 = 1.333 V_2$$

$$Z_{12} = \left. \frac{V_1}{I_2} \right|_{I_1=0} = \frac{-I_3}{I_2} = \frac{0.333 V_2}{1.333 V_2} = 0.25 \Omega$$

$$Z_{22} = \left. \frac{V_2}{I_2} \right|_{I_1=0} = \frac{V_2}{1.333 V_2} = 0.75 \Omega$$



Q. Define Y-parameters (Admittance parameters) (Short Circuit admittance parameters)

Sol: These are also called admittance parameters. These are obtained by expressing currents at two ports in terms of voltages at two ports. Thus, voltages V_1 & V_2 are independent variables, while I_1 & I_2 are dependent variables. We have

$$I_1 = Y_{11} V_1 + Y_{12} V_2$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2$$

In matrix form,
$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

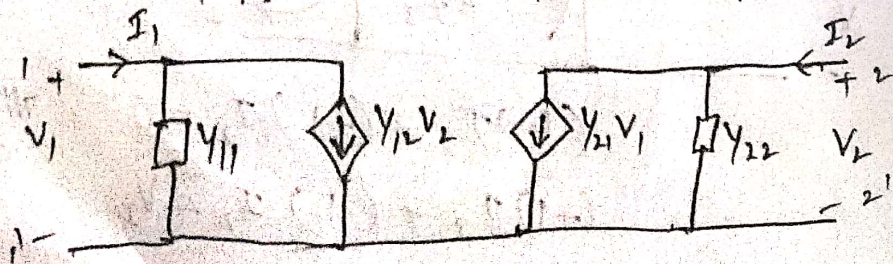
$$Y_{11} = \left. \frac{I_1}{V_1} \right|_{V_2=0} \quad (w)$$

$$Y_{12} = \left. \frac{I_1}{V_2} \right|_{V_1=0} \quad (w)$$

$$Y_{21} = \left. \frac{I_2}{V_1} \right|_{V_2=0} \quad (w)$$

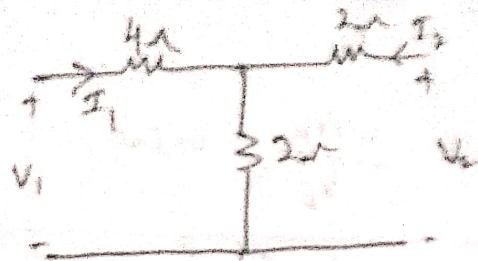
$$Y_{22} = \left. \frac{I_2}{V_2} \right|_{V_1=0} \quad (w)$$

These parameters are defined individually only when the voltage in any one of the ports is zero. This corresponds to the condition that one of the ports is short circuited. Hence Y-parameters are also called short circuit admittance parameters.

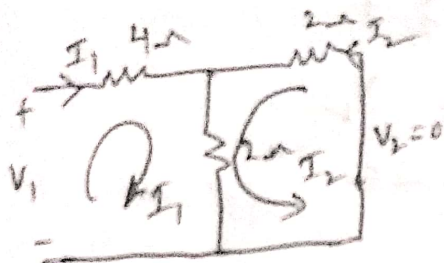


Equivalent network of a two port n/w in terms of Y-parameters

Determine the admittance parameters of the T network shown in figure



Sol: $V_2 = 0$



$$(6) I_1 + (4) I_2 = V_1$$

$$(4) I_1 + (4) I_2 = 0$$

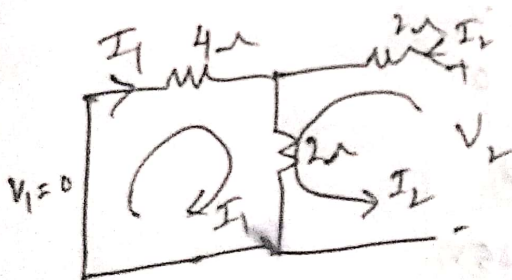
$$I_1 = 0.2 V_1$$

$$I_2 = -0.1 V_1$$

$$Y_{11} = \frac{I_1}{V_1} \bigg|_{V_2=0} = \frac{0.2 V_1}{V_1} = 0.2 \text{ S}$$

$$Y_{21} = \frac{I_2}{V_1} \bigg|_{V_2=0} = \frac{-0.1 V_1}{V_1} = -0.1 \text{ S}$$

$V_1 = 0$



$$(6) I_1 + (4) I_2 = 0$$

$$(4) I_1 + (4) I_2 = V_2$$

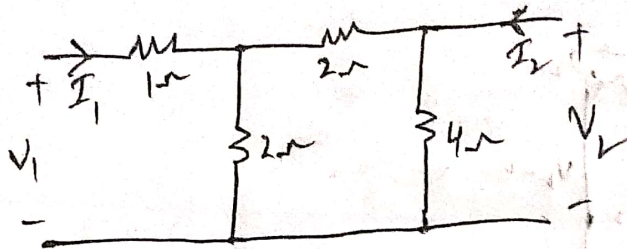
$$I_1 = -0.1 V_2$$

$$I_2 = 0.3 V_2$$

$$Y_{12} = \frac{I_1}{V_2} \Big|_{V_1=0} = \frac{-0.1V_2}{V_2} = -0.1 \Omega$$

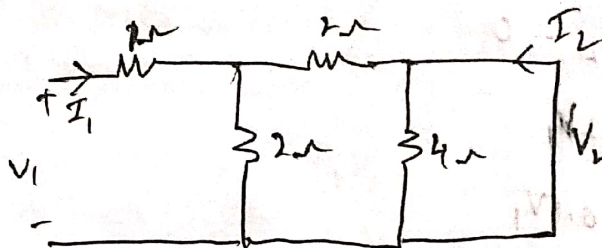
$$Y_{22} = \frac{I_2}{V_2} \Big|_{V_1=0} = \frac{0.3V_2}{V_2} = 0.3 \Omega$$

Q. Find the Y-parameters for the network shown in figure.

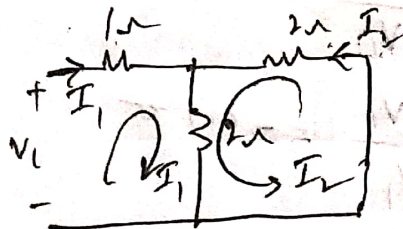


Sol:

$$V_2 = 0$$



Remove 4Ω
(Short is across 4Ω)



$$(3)I_1 + (4)I_2 = V_1$$

$$(4)I_1 + (4)I_2 = 0$$

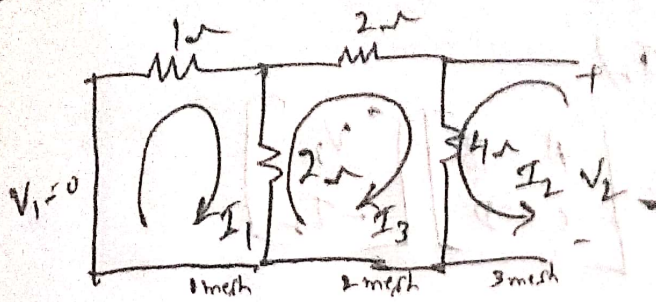
$$I_1 = 0.5V_1$$

$$I_2 = -0.25V_1$$

$$Y_{11} = \frac{I_1}{V_1} \Big|_{V_2=0} = \frac{0.5V_1}{V_1} = 0.5 \Omega$$

$$Y_{21} = \frac{I_2}{V_1} \Big|_{V_2=0} = \frac{-0.25V_1}{V_1} = -0.25 \Omega$$

$$V_1 = 0$$



$$(3)I_1 + (-2)I_3 + (0)I_2 = 0$$

$$(-2)I_1 + (8)I_3 + (+4)I_2 = 0$$

$$(0)I_1 + (+4)I_3 + (4)I_2 = V_2$$

$$I_1 = -0.25V_2$$

$$I_3 = -0.375V_2$$

$$I_2 = 0.625V_2$$

$$Y_{12} = \frac{I_1}{V_2} \Big|_{V_1=0} = \frac{-0.25V_2}{V_2} = -0.25\Omega$$

$$Y_{22} = \frac{I_2}{V_2} \Big|_{V_1=0} = \frac{0.625V_2}{V_2} = 0.625\Omega$$

Q. Define ABCD parameters (Transmission parameters) (chain parameters)

Sol: These parameters are known as transmission parameters. These are generally used in the analysis of power transmission in which the input port is referred as the sending end and the output port is referred as receiving end.

These are obtained by expressing voltage V_1 & current I_1 at input port in terms of voltage V_2 and current I_2 at output port. Thus, voltage V_2 & current I_2 are independent variables while voltage V_1 & current I_1 are dependent variables. Thus we have,

$$V_1 = AV_2 + B(-I_2)$$

$$I_1 = CV_2 + D(-I_2)$$

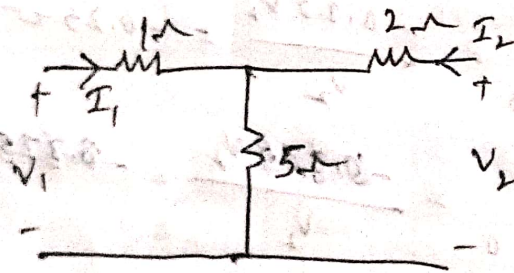
In matrix form,
$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

$$A = \left. \frac{V_1}{V_2} \right|_{I_2=0} \quad B = \left. \frac{V_1}{-I_2} \right|_{V_2=0}$$

$$C = \left. \frac{I_1}{V_2} \right|_{I_2=0} \quad D = \left. \frac{I_1}{-I_2} \right|_{V_2=0}$$

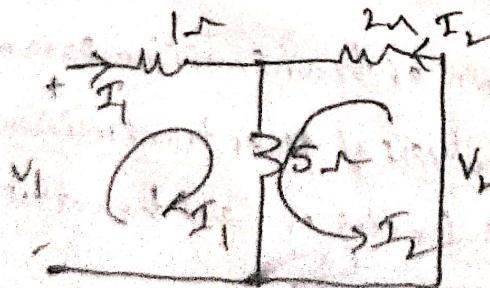
The equivalent circuit of a two port network is not possible in terms of T -parameters.

Q. Find the transmission parameters for the circuit shown in figure.



Sol:

$$\underline{V_2 = 0}$$



$$(6)I_1 + (+5)I_2 = V_1$$

$$(45)I_1 + (7)I_2 = 0$$

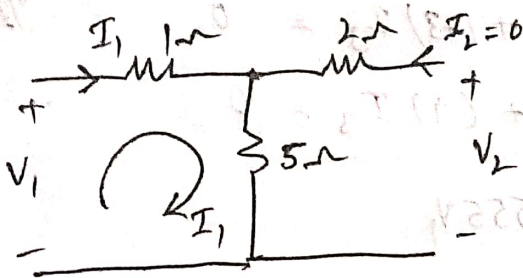
$$I_1 = 0.411V_1$$

$$I_2 = -0.294V_1$$

$$B = \left. \frac{V_1}{-I_2} \right|_{V_2=0} = \frac{V_1}{0.2941V_1} = 3.4 \Omega$$

$$D = \left. \frac{I_1}{-I_2} \right|_{V_2=0} = \frac{0.411V_1}{0.2941V_1} = 1.4$$

$$\underline{I_2=0}$$

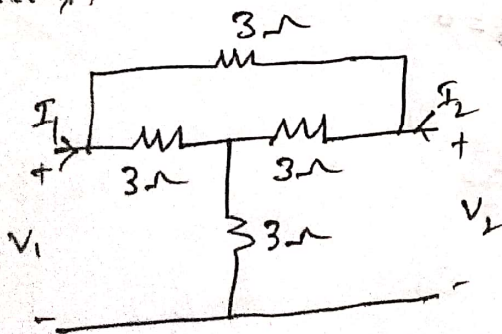


$$I_1 = \frac{V_1}{(1+5)} = \frac{V_1}{6}$$

$$A = \left. \frac{V_1}{V_2} \right|_{I_2=0} = \frac{V_1}{5I_1} = \frac{V_1}{5 \times \frac{V_1}{6}} = 1.2 \quad V_2 = 5I_1$$

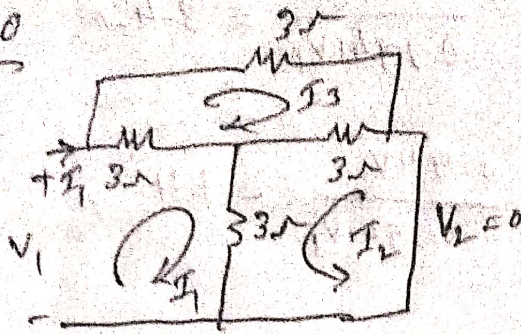
$$C = \left. \frac{I_1}{V_2} \right|_{I_2=0} = \frac{I_1}{5I_1} = 0.2 \text{ S}$$

Q. Refer the bridge circuit shown in the figure. Find the transmission parameters.



Q11:

$$V_2 = 0$$



$$(6)I_1 + (+3)I_2 + (-3)I_3 = V_1$$

$$(+3)I_1 + (6)I_2 + (+3)I_3 = 0$$

$$(-3)I_1 + (+3)I_2 + (9)I_3 = 0$$

Note the sign for
off-diagonal
elements

$$I_1 = 0.555V_1$$

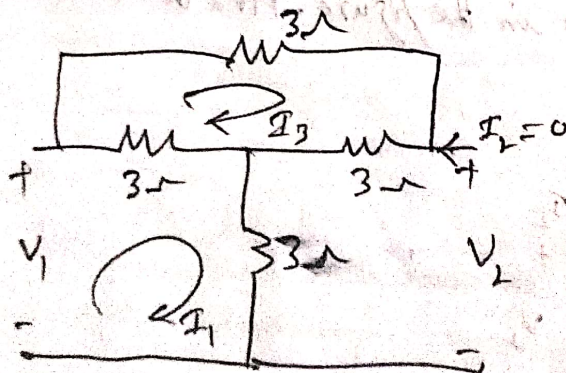
$$I_2 = -0.444V_1$$

$$I_3 = 0.333V_1$$

$$B = \frac{V_1}{-I_2} \bigg|_{V_2=0} = \frac{V_1}{0.444V_1} = 2.25 \Omega$$

$$D = \frac{I_1}{-I_2} \bigg|_{V_2=0} = \frac{0.555V_1}{0.444V_1} = 1.25$$

$$I_2 = 0$$



$$(6)I_1 + (-3)I_3 = V_1$$

$$(-3)I_1 + (9)I_3 = 0$$

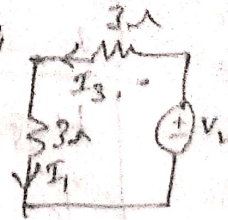
$$I_1 = 0.2V_1$$

$$I_3 = 0.066V_1$$



$$A = \frac{V_1}{V_2} = \frac{V_1}{3I_1 + 3I_3} = \frac{V_1}{3 \times 0.2V_1 + 3 \times 0.066V_1}$$

$$A = \frac{V_1}{V_1(3 \times 0.2 + 3 \times 0.066)} = 1.25$$



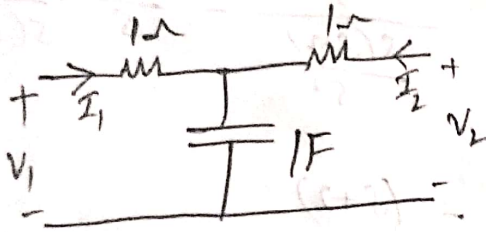
$$3I_3 + 3I_1 - V_2 = 0$$

$$V_2 = 3I_1 + 3I_3$$

$$C = \frac{I_1}{V_2} \Big|_{I_2=0} = \frac{I_1}{3I_1 + 3I_3} = \frac{0.2V_1}{3 \times 0.2V_1 + 3 \times 0.066V_1}$$

$$C = 0.25 \text{ V}$$

6. Determine the transmission parameters in the S-domain for the network shown in the figure.

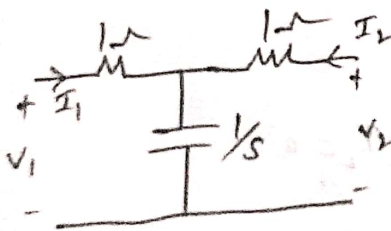


In S-domain : $L \rightarrow SL \rightarrow S \times 1 = 1S$

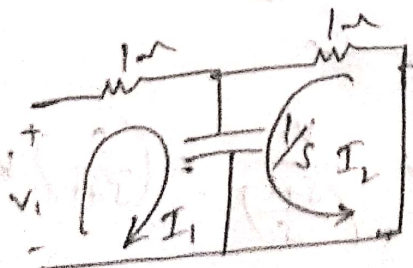
If $L = 1H$
 $C = 1F$

$$C \rightarrow \frac{1}{s} \rightarrow \frac{1}{1 \times s} = \frac{1}{s}$$

If $L = 2H \rightarrow 2S$
 $C = 2F \rightarrow \frac{1}{2S}$



$$V_2 = 0$$



$$(1 + 1/s)I_1 + (1/s)I_2 = V_1$$

$$(1/s)I_1 + (1 + 1/s)I_2 = 0$$

$$I_1 = \frac{\begin{vmatrix} V_1 & 1/s \\ a & 1+1/s \end{vmatrix}}{\begin{vmatrix} 1+1/s & 1/s \\ 1/s & 1+1/s \end{vmatrix}} = \frac{V_1 \left(\frac{s+1}{s} \right)}{\left(\frac{s+1}{s} \right)^2 - \frac{1}{s^2}}$$

$$I_1 = \frac{V_1 \left(\frac{s+1}{s} \right)}{\frac{s^2+2s+1}{s^2} - \frac{1}{s^2}} = \frac{V_1 \left(\frac{s+1}{s} \right)}{\frac{s^2+2s+1-1}{s^2}} = V_1 \left(\frac{s+1}{s} \right) \times \frac{s^2}{s(s+2)}$$

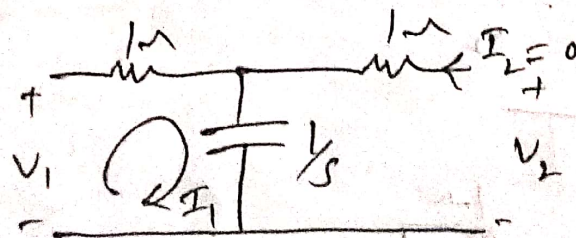
$$I_1 = \frac{(s+1)}{(s+2)} V_1$$

$$I_2 = \frac{\begin{vmatrix} 1+1/s & V_1 \\ 1/s & 0 \end{vmatrix}}{\frac{s^2+2s}{s^2}} = \frac{-\frac{V_1}{s}}{\frac{s(s+2)}{s^2}} = -\frac{V_1}{s} \times \frac{s^2}{s(s+2)} = -\frac{1}{(s+2)}$$

$$B = \frac{V_1}{-I_2} \Big|_{V_2=0} = \frac{V_1}{\left(\frac{1}{s+2} \right) V_1} = (s+2)$$

$$D = \frac{I_1}{-I_2} \Big|_{V_2=0} = \frac{(s+1)}{(s+2)} \times \frac{(s+2)}{1} = s+1$$

$$I_2 = 0$$

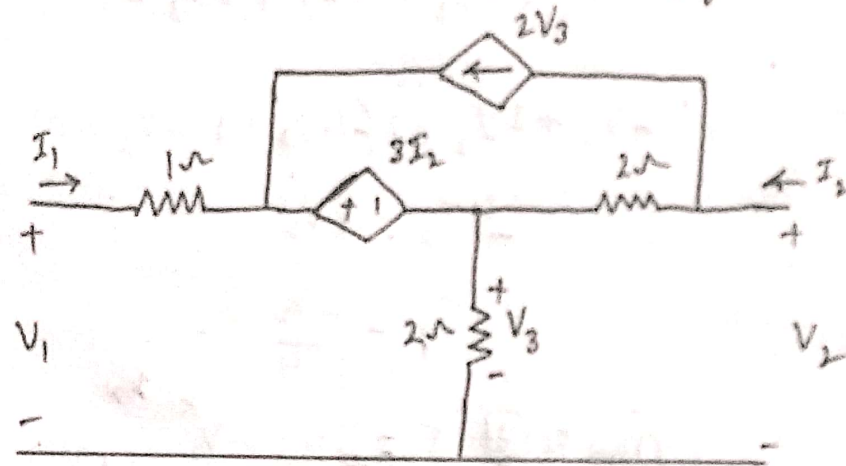


$$I_1 = \frac{V_1}{1+1/s} = \frac{V_1}{\left(\frac{s+1}{s} \right)} = \left(\frac{s}{s+1} \right) V_1$$

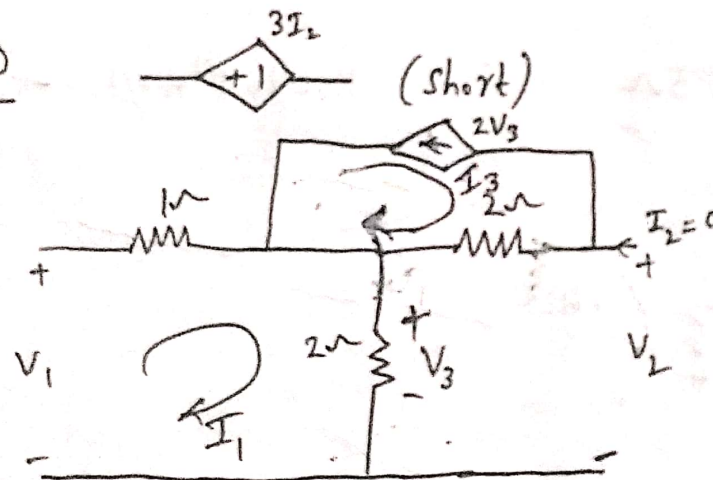
$$A = \frac{V_1}{V_2} \Big|_{I_2=0} = \frac{V_1}{\frac{1}{s} I_1} = \frac{V_1}{\frac{1}{s} \times \left(\frac{s}{s+1} \right) V_1} = (s+1) \quad V_2 = \frac{1}{s} I_1$$

$$C = \frac{I_1}{V_2} \Big|_{I_2=0} = \frac{I_1}{\frac{1}{s} I_1} = s$$

For the two port network shown in the figure below find Z parameters



$I_2 = 0$



Control Variable
 $V_3 = +2I_1$

KCL to Non essential mesh: $I_3 = -2V_3 = -2(+2I_1) = -4I_1$ - ①

KVL to essential mesh: $-V_1 + 1I_1 + 2I_1 = 0$ - ②

$3I_1 = V_1$

$I_1 = \frac{V_1}{3}$, $I_3 = -\frac{4V_1}{3}$

$$Z_{11} = \left. \frac{V_1}{I_1} \right|_{I_2=0} = \frac{V_1}{\frac{V_1}{3}} = 3\Omega$$

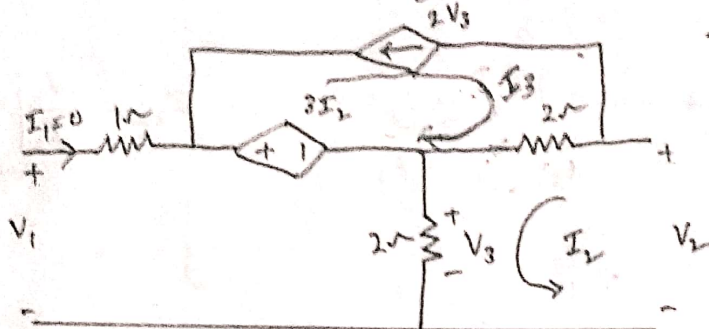
$$Z_{21} = \left. \frac{V_2}{I_1} \right|_{I_2=0} = \frac{2I_1 + 2I_3}{I_1}$$

$$\text{KVL to mesh 2: } -V_2 + 2I_3 + 2I_1 = 0$$

$$V_2 = 2I_1 + 2I_3$$

$$= \frac{2 \times \frac{V_1}{3} + 2 \times \frac{-4V_1}{3}}{\frac{V_1}{3}} = \frac{\frac{V_1}{3} (2-8)}{\frac{V_1}{3}} = -6\Omega$$

$$I_1 = 0$$



$$V_3 = 2I_2$$

$$\text{KCL to nonessential mesh: } I_3 = -2V_3 = -2(+2I_2) = -4I_2 \quad \text{--- (3)}$$

$$\text{KVL to essential mesh: } -V_2 + 2(I_2 + I_3) + 2I_2 = 0$$

$$-V_2 + 2I_2 + 2I_3 + 2I_2 = 0$$

$$-V_2 + 2I_2 + 2(-4I_2) + 2I_2 = 0 \quad \text{from (3)}$$

$$-4I_2 = +V_2$$

$$I_2 = \frac{-V_2}{4}$$

$$\text{from eq (3)} \quad I_3 = -4 \times \frac{-V_2}{4} = +V_2$$

$$Z_{12} = \left. \frac{V_1}{I_2} \right|_{I_1=0} = \frac{5I_2}{I_2} = 5\Omega$$

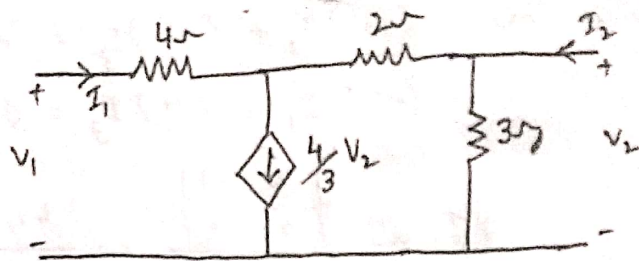
$$\text{KVL to 1st mesh: } -V_1 + 3I_2 + 2I_2 = 0$$

$$V_1 = 5I_2$$

$$Z_{22} = \left. \frac{V_2}{I_2} \right|_{I_1=0} = \frac{V_2}{\frac{-V_2}{4}} = -4\Omega$$

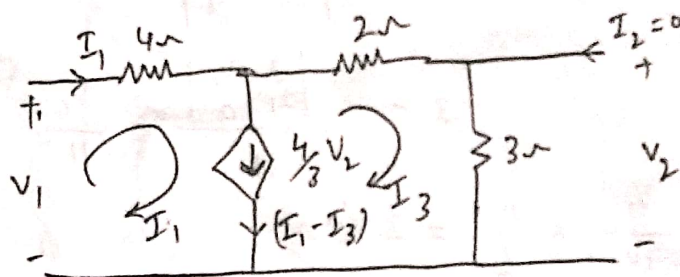
$$Z = \begin{bmatrix} 3 & 5 \\ -6 & -4 \end{bmatrix}$$

Prob 5: Determine Z and Y parameters for the two port network shown in the figure



Sol:

$$\underline{I_2 = 0}$$



KCL to supermesh: $I_1 - I_3 = \frac{4}{3} V_2$ But $V_2 - 3I_3 = 0$
 $I_1 - I_3 = \frac{4}{3} (3I_3)$ $V_2 = 3I_3$

$$I_1 - I_3 = 4I_3$$

$$I_1 - 5I_3 = 0 \quad \text{--- (1)}$$

KVL to supermesh: $-V_1 + 4I_1 + 2I_3 + 3I_3 = 0$

$$+4I_1 + 5I_3 = +V_1 \quad \text{--- (2)}$$

(From equations 1 & 2)

Use calculator
 $a_1 = 1, b_1 = -5, c_1 = 0$
 $a_2 = 4, b_2 = 5, c_2 = 1$
 Attach V_1 for
 answer

$$(1) I_1 + (-5)I_3 = 0$$

$$(4) I_1 + (5)I_3 = V_1$$

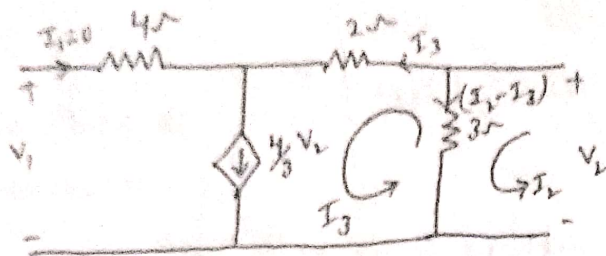
$$I_1 = 0.2V_1$$

$$I_3 = 0.04V_1$$

$$Z_{11} = \frac{V_1}{I_1} \Big|_{I_2=0} = \frac{V_1}{0.2V_1} = 5\Omega$$

$$Z_{12} = \frac{V_2}{I_1} \Big|_{I_2=0} = \frac{3I_3}{I_1} = \frac{3 \times 0.04V_1}{0.2V_1} = 0.6\Omega$$

$$I_1 = 0$$



KVL eq. to nonessential mesh: $I_3 = \frac{4}{3}V_2$

But $-V_2 + 3(I_2 - I_3) = 0$

$$V_2 = 3I_2 - 3I_3$$

$$I_3 = \frac{4}{3}(3I_2 - 3I_3)$$

$$I_3 = 4I_2 - 4I_3$$

$$4I_2 - 5I_3 = 0 \quad \text{--- (1)}$$

KVL to essential mesh: $V_2 - 3(I_2 - I_3) = 0$

$$+3I_2 - 3I_3 = V_2 \quad \text{--- (2)}$$

Solving (1) & (2) $(4)I_2 + (-5)I_3 = 0$

$$(3)I_2 + (-3)I_3 = V_2$$

$$I_2 = 1.66V_2 \quad I_3 = 1.33V_2$$

$$Z_{12} = \frac{V_2}{I_1} \Big|_{I_1=0} = \frac{3I_2 - 5I_3}{I_2}$$

$$= \frac{3 \times 1.66V_2 - 5 \times 1.33V_2}{1.66V_2}$$

$$= \frac{V_2(3 \times 1.66 - 5 \times 1.33)}{1.66V_2}$$

$$= -1\Omega$$

$$-V_1 - 2I_3 + 3(I_2 - I_3) = 0$$

$$V_1 = -2I_3 + 3I_2 - 3I_3$$

$$V_1 = 3I_2 - 5I_3$$

$$Z_{22} = \frac{V_2}{I_2} \Big|_{I_1=0} = \frac{V_2}{1.66V_2} = 0.6\Omega$$

$$Z = \begin{bmatrix} 5 & -1 \\ 0.6 & 0.6 \end{bmatrix}$$

$$Y = [Z]^{-1} = \frac{\text{adj } Z}{|Z|} = \frac{\begin{bmatrix} 0.6 & +1 \\ -0.6 & 5 \end{bmatrix}}{5 \times 0.6 + 1 \times 0.6}$$

$$Y = \frac{\begin{bmatrix} 0.6 & +1 \\ -0.6 & 5 \end{bmatrix}}{3.6}$$

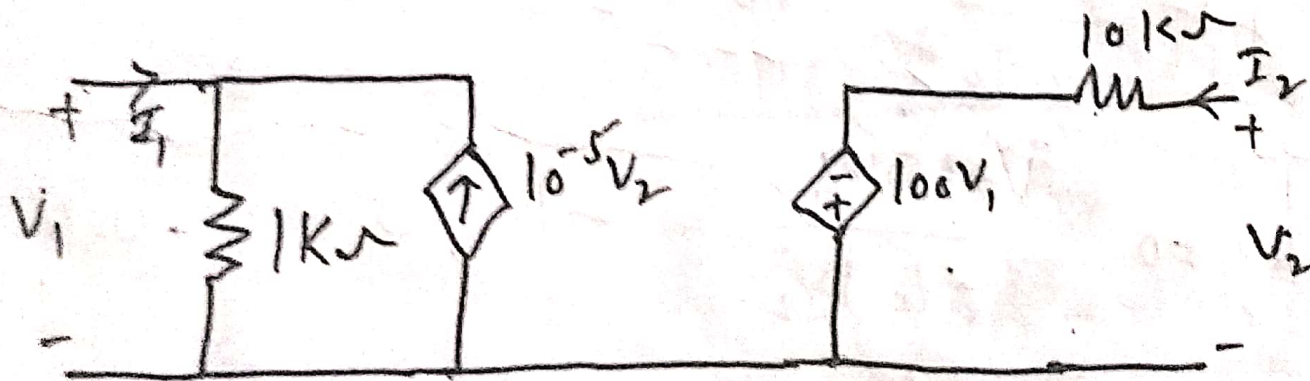
$$Y = \begin{bmatrix} 0.1666 & 0.2777 \\ -0.1666 & 1.3888 \end{bmatrix}$$

(36)
To get adj Z : Interchange the diagonal elements of $[Z]$ & change the signs of off-diagonal elements of $[Z]$

or directly use calculator to obtain $[Z]^{-1}$

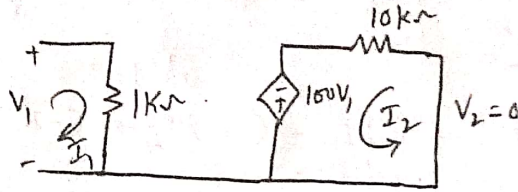
Find the inverse of the matrix $Z = \begin{bmatrix} 0.6 & 1 \\ -0.6 & 5 \end{bmatrix}$

Find Y parameters for the network shown in the figure



Sol: $V_2 = 0$

$10^{-5} V_2 = 0$ (open)



KVL eqs to two essential meshes: $-V_1 + 1 \times 10^3 I_1 = 0$

$I_1 = \frac{V_1}{10^3} = 0.001 V_1$

$+10 \times 10^3 I_2 - 100 V_1 = 0$

$I_2 = \frac{100 V_1}{10 \times 10^3} = 0.01 V_1$

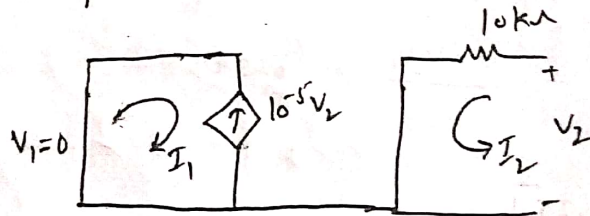
$Y_{11} = \frac{I_1}{V_1} \Big|_{V_2=0} = \frac{0.001 V_1}{V_1} = 0.001 \text{ } \Omega^{-1}$

$Y_{21} = \frac{I_2}{V_1} \Big|_{V_2=0} = \frac{0.01 V_1}{V_1} = 0.01 \text{ } \Omega^{-1}$

$V_1 = 0$

$100 V_1 = 0$ (short)

Also due to short across $1 \text{ K}\Omega$ when $V_2 = 0$, $1 \text{ K}\Omega$ can be neglected.



KCL: $I_1 = -10^{-5} V_2$

KVL: $-V_2 + 10 \times 10^3 I_2 = 0$

$I_2 = \frac{V_2}{10 \times 10^3} = 10^{-4} V_2$

Caution: I_1 should always be clockwise

I_2 should always be anticlockwise

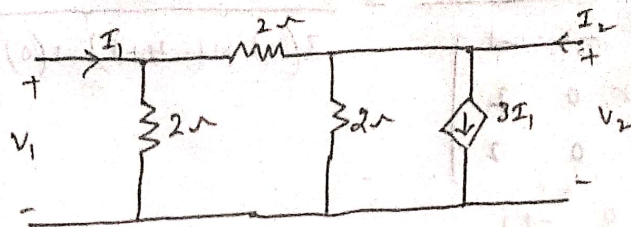
$Y_{12} = \frac{I_1}{V_2} \Big|_{V_1=0} = \frac{-10^{-5} V_2}{V_2} = -10^{-5} \text{ } \Omega^{-1} = -0.00001 \text{ } \Omega^{-1}$

$Y_{22} = \frac{I_2}{V_2} \Big|_{V_1=0} = \frac{10^{-4} V_2}{V_2} = 10^{-4} \text{ } \Omega^{-1} = 0.0001 \text{ } \Omega^{-1}$

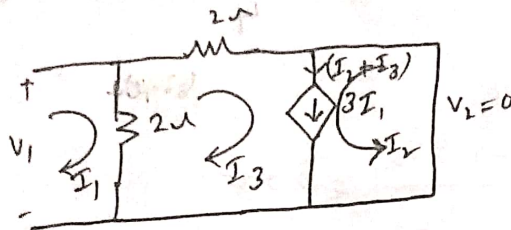
$Y = \begin{bmatrix} 0.001 & -0.00001 \\ 0.01 & 0.0001 \end{bmatrix} \text{ } \Omega^{-1}$

Prob 8: Determine Y parameters of the two port network shown in the figure.

39



Sol: $V_2 = 0$. A short is across 2Ω . It can be neglected.



$$\text{KCL equation to supermesh: } I_2 + I_3 = 3I_1 \Rightarrow (3)I_1 + (-1)I_2 + (-1)I_3 = 0 \quad \text{--- (1)}$$

$$\text{KVL equation to supermesh: } 2(I_3 - I_1) + 2I_3 = 0 \Rightarrow (-2)I_1 + (0)I_2 + (4)I_3 = 0 \quad \text{--- (2)}$$

$$\text{KVL equation to essential mesh: } V_1 + 2(I_1 - I_3) = 0 \Rightarrow (+2)I_1 + (0)I_2 + (-2)I_3 = +V_1 \quad \text{--- (3)}$$

Solving,

$$I_1 = 1V_1$$

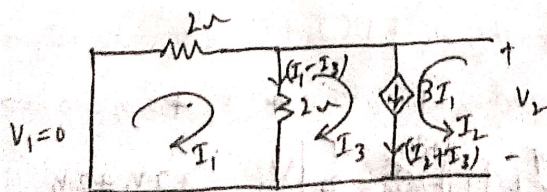
$$I_2 = 2.5V_1$$

$$I_3 = 0.5V_1$$

$$Y_{11} = \frac{I_1}{V_1} \Big|_{V_2=0} = \frac{V_1}{V_1} = 1\Omega$$

$$Y_{21} = \frac{I_2}{V_1} \Big|_{V_2=0} = \frac{2.5V_1}{V_1} = 2.5\Omega$$

$V_1 = 0$. A short is across 2Ω . It can be neglected.



$$\text{KCL eq. to supermesh: } I_2 + I_3 = 3I_1 \Rightarrow (3)I_1 + (-1)I_2 + (-1)I_3 = 0 \quad \text{--- (1)}$$

$$\text{KVL eq to supermesh: } -V_2 + 2(I_1 - I_3) = 0 \Rightarrow (+2)I_1 + (0)I_2 + (-2)I_3 = +V_2 \quad \text{--- (2)}$$

$$\text{KVL eq to essential mesh: } +2I_1 + 2(I_1 - I_3) = 0 \Rightarrow (+4)I_1 + (0)I_2 + (-2)I_3 = 0 \quad \text{--- (3)}$$

Solving,

$$I_1 = -0.5V_2$$

$$I_2 = -0.5V_2$$

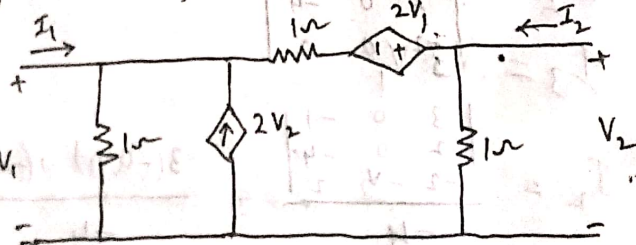
$$I_3 = -1V_2$$

$$Y_{12} = \frac{I_1}{V_2} \Big|_{V_1=0} = \frac{-0.5V_2}{V_2} = -0.5 \Omega^{-1}$$

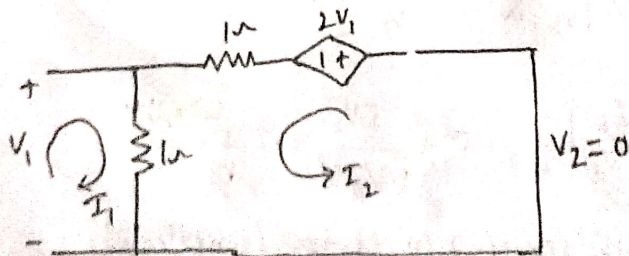
$$Y_{22} = \frac{I_2}{V_2} \Big|_{V_1=0} = \frac{-0.5V_2}{V_2} = -0.5 \Omega^{-1}$$

$$Y = \begin{bmatrix} 1 & -0.5 \\ 2.5 & -0.5 \end{bmatrix} \Omega^{-1}$$

Prob 9: Find the Y parameters for the network shown in the figure.



Sol: $V_2 = 0$ $2V_2 = 0$ (open). 1Ω connected across short can be eliminated.



$$\text{KVL to essential mesh: } -V_1 + 1(I_1 + I_2) = 0 \Rightarrow I_1 + I_2 = +V_1 \quad \text{--- (1)}$$

$$\text{KVL to essential mesh: } -1(I_1 + I_2) - 1I_2 - 2V_1 = 0 \Rightarrow -I_1 - 2I_2 = +2V_1 \quad \text{--- (2)}$$

Solving (1) & (2)

$$(1) I_1 + (1) I_2 = V_1$$


$$(-1) I_1 + (-2) I_2 = 2V_1$$

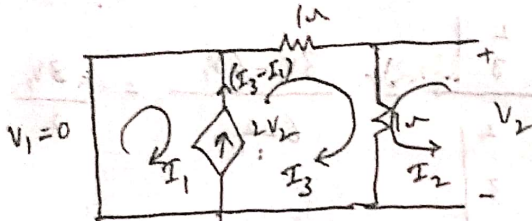
$$I_1 = 4V_1$$

$$I_2 = -3V_1$$

$$Y_{11} = \frac{I_1}{V_1} \Big|_{V_2=0} = \frac{4V_1}{V_1} = 4 \Omega$$

$$Y_{21} = \frac{I_2}{V_1} \Big|_{V_2=0} = \frac{-3V_1}{V_1} = -3 \Omega$$

$V_1=0$  = 0 (short). 1Ω can be neglected since short is across it.



KCL to super mesh: $I_3 - I_1 = 2V_2 \Rightarrow (-1)I_1 + (0)I_2 + (1)I_3 = 2V_2$ - (1)

KVL to super mesh: $+1I_3 + 1(I_2 + I_3) = 0 \Rightarrow (0)I_1 + (1)I_2 + (2)I_3 = 0$ - (2)

KVL to essential mesh: $-V_2 + 1(I_2 + I_3) = 0 \Rightarrow (0)I_1 + (1)I_2 + (1)I_3 = +V_2$ - (3)

Solving,

$$I_1 = -3V_2$$

$$I_2 = 2V_2$$

$$I_3 = -1V_2$$

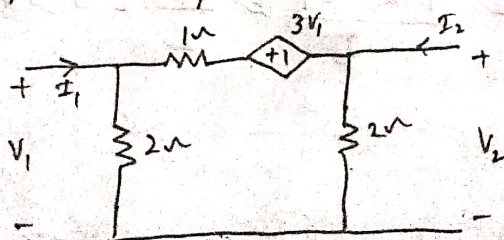
$$Y_{12} = \frac{I_1}{V_2} \Big|_{V_1=0} = \frac{-3V_2}{V_2} = -3 \Omega$$

$$Y_{22} = \frac{I_2}{V_2} \Big|_{V_1=0} = \frac{2V_2}{V_2} = 2 \Omega$$

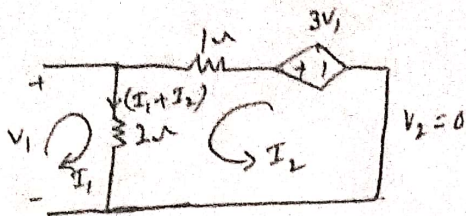
$$Y = \begin{bmatrix} 4 & -3 \\ -3 & 2 \end{bmatrix} \Omega$$

Prob 10:

Find the Y parameters for the circuit shown in the figure.



$V_2 = 0$. 2Ω across short can be eliminated.



KVL to essential mesh: $-V_1 + 2(I_1 + I_2) = 0 \Rightarrow (+2)I_1 + (+2)I_2 = +V_1$ -①

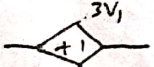
KVL to essential mesh: $-3V_1 + 1I_2 + 2(I_1 + I_2) \Rightarrow (+2)I_1 + (+3)I_2 = +3V_1$ -②

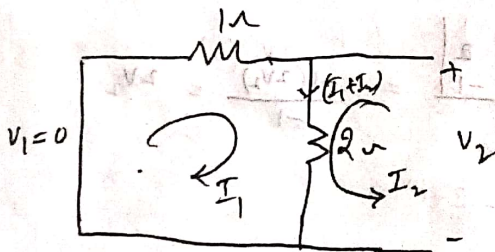
Solving, $I_1 = -1.5V_1$

$I_2 = 2V_1$

$Y_{11} = \frac{I_1}{V_1} \Big|_{V_2=0} = \frac{-1.5V_1}{V_1} = -1.5\Omega$

$Y_{21} = \frac{I_2}{V_1} \Big|_{V_2=0} = \frac{2V_1}{V_1} = 2\Omega$

$V_1 = 0$  = 0 (short). 2Ω across short can be neglected.



KVL equations to two essential meshes: $(3)I_1 + (+2)I_2 = 0$
(By inspection)

$(+2)I_1 + (2)I_2 = +V_2$

Solving,

$I_1 = -1V_2$

$I_2 = 1.5V_2$

$Y_{12} = \frac{I_1}{V_2} \Big|_{V_1=0} = \frac{-1V_2}{V_2} = -1\Omega$

$Y_{22} = \frac{I_2}{V_2} \Big|_{V_1=0} = \frac{1.5V_2}{V_2} = 1.5\Omega$

$Y = \begin{bmatrix} -1.5 & -1 \\ 2 & 1.5 \end{bmatrix}$